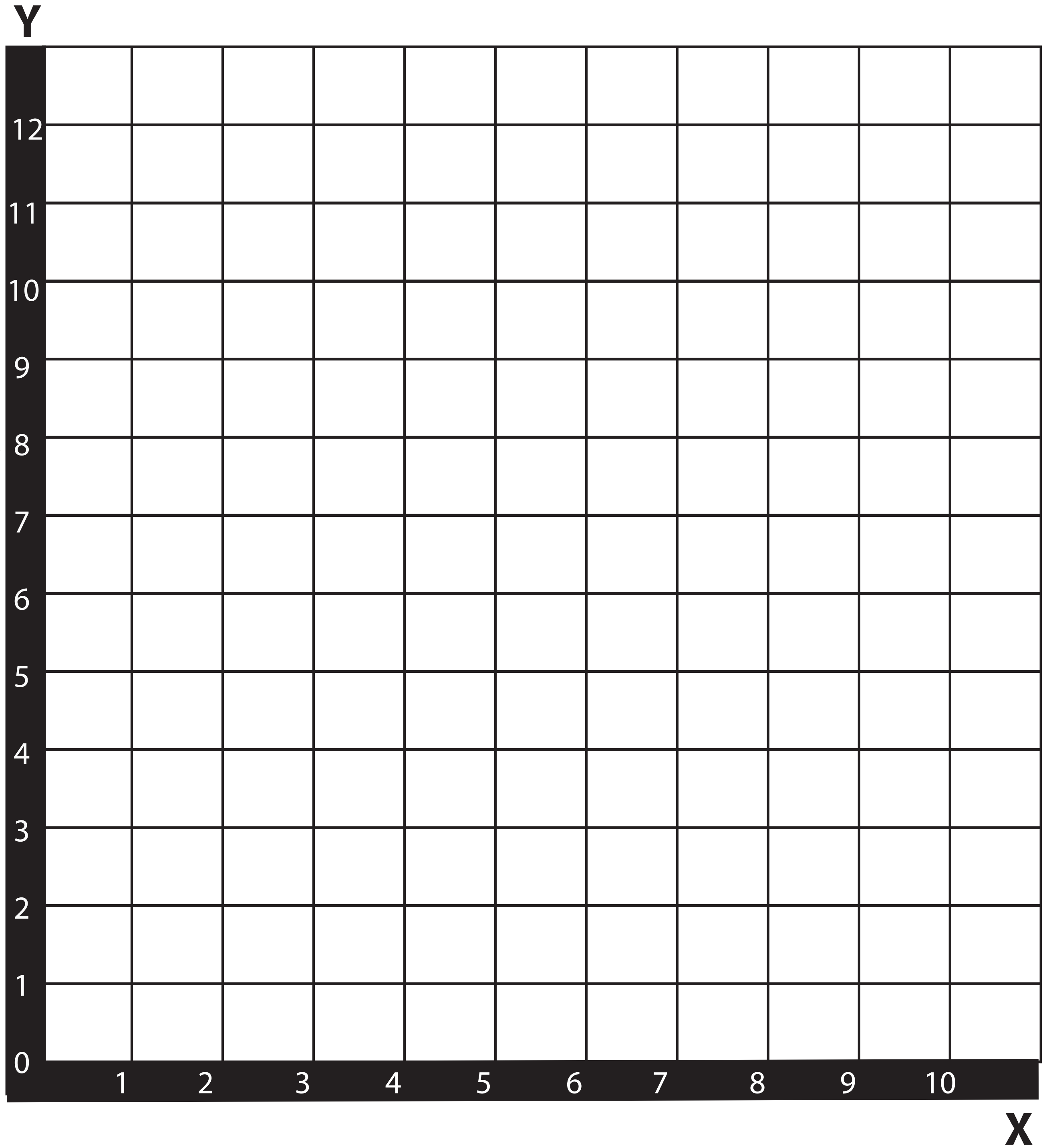
X · Y

Y \ X	1	2	3	4	5	6	7	8	9	10
10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	63	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10

X•Y

Connect the Dots

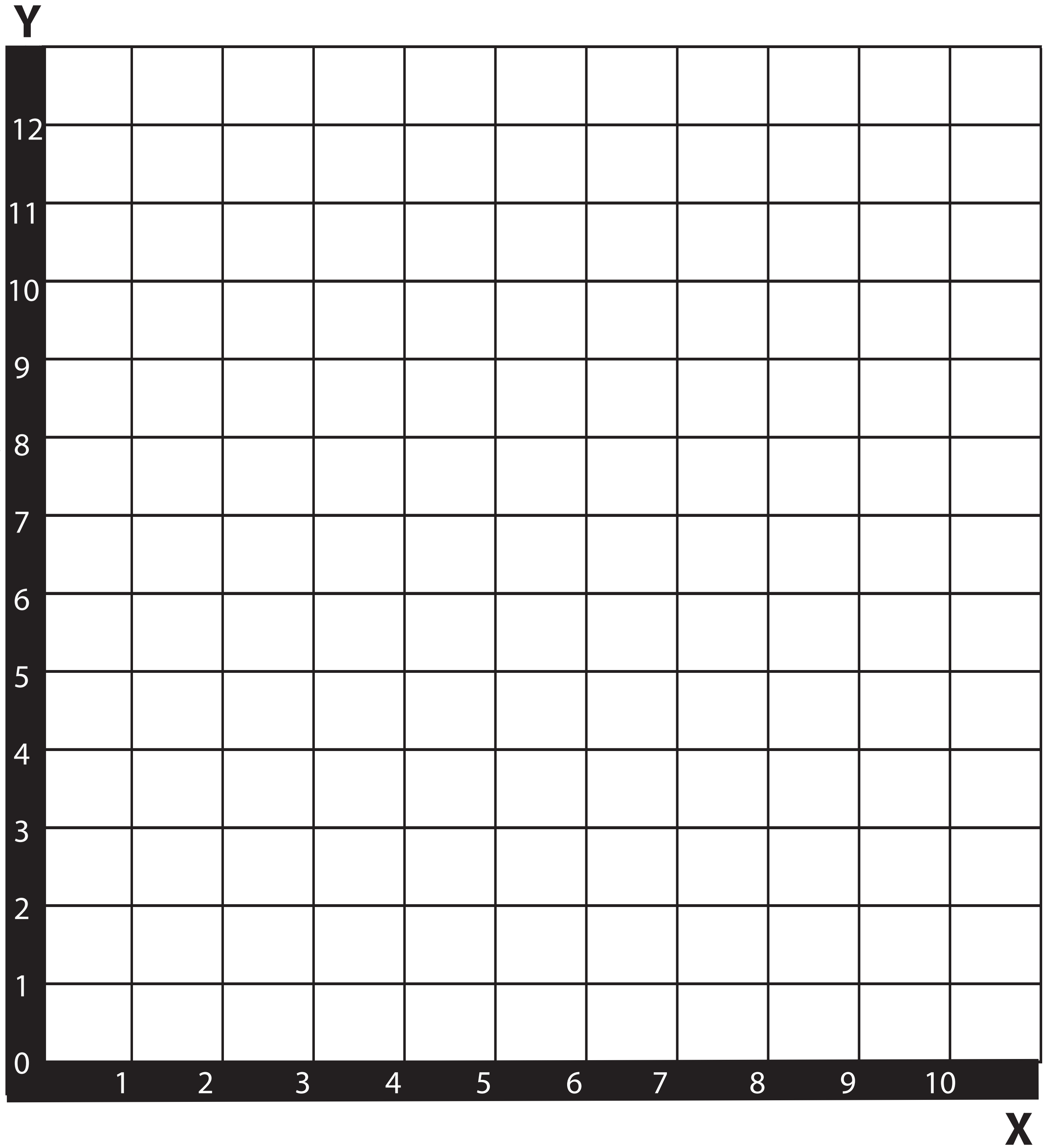


Over to X, up to Y, draw dot and Connect

(X,Y) (X,Y) (X,Y) (X,Y)

X•Y

Connect the Dots

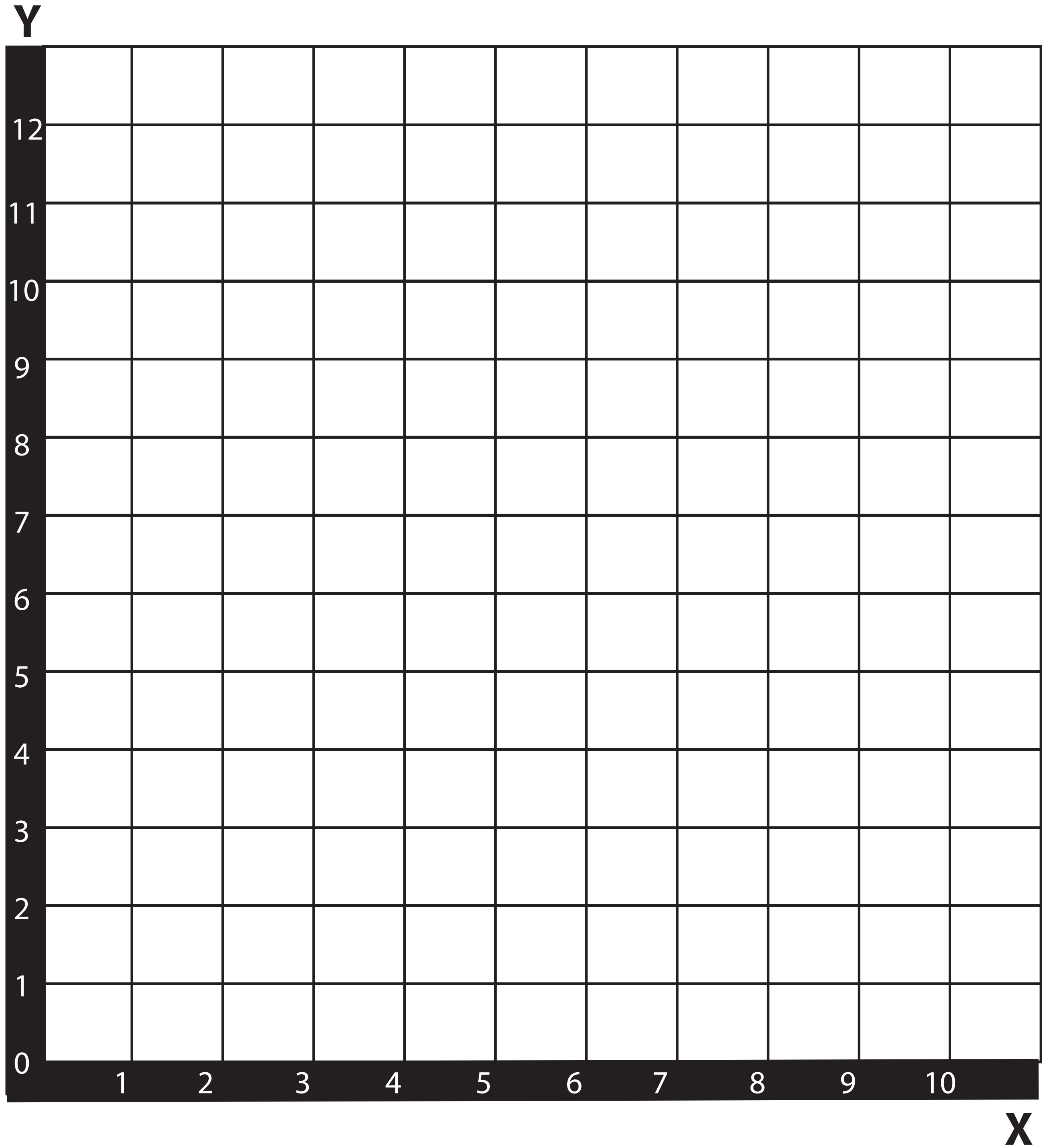


Over to X, up to Y, draw dot and connect.

(X,Y)	(X,Y)	(X,Y)	(X,Y)
(2,1)	(3,8)	(6,6)	(2,1)
(1,3)	(4,9)	(6,5)	
(2,5)	(5,9)	(7,4)	
(3,5)	(6,8)	(8,3)	
(4,5)	(6,7)	(8,2)	
(3,7)	(7,6)	(7,1)	

X•Y

Connect the Dots

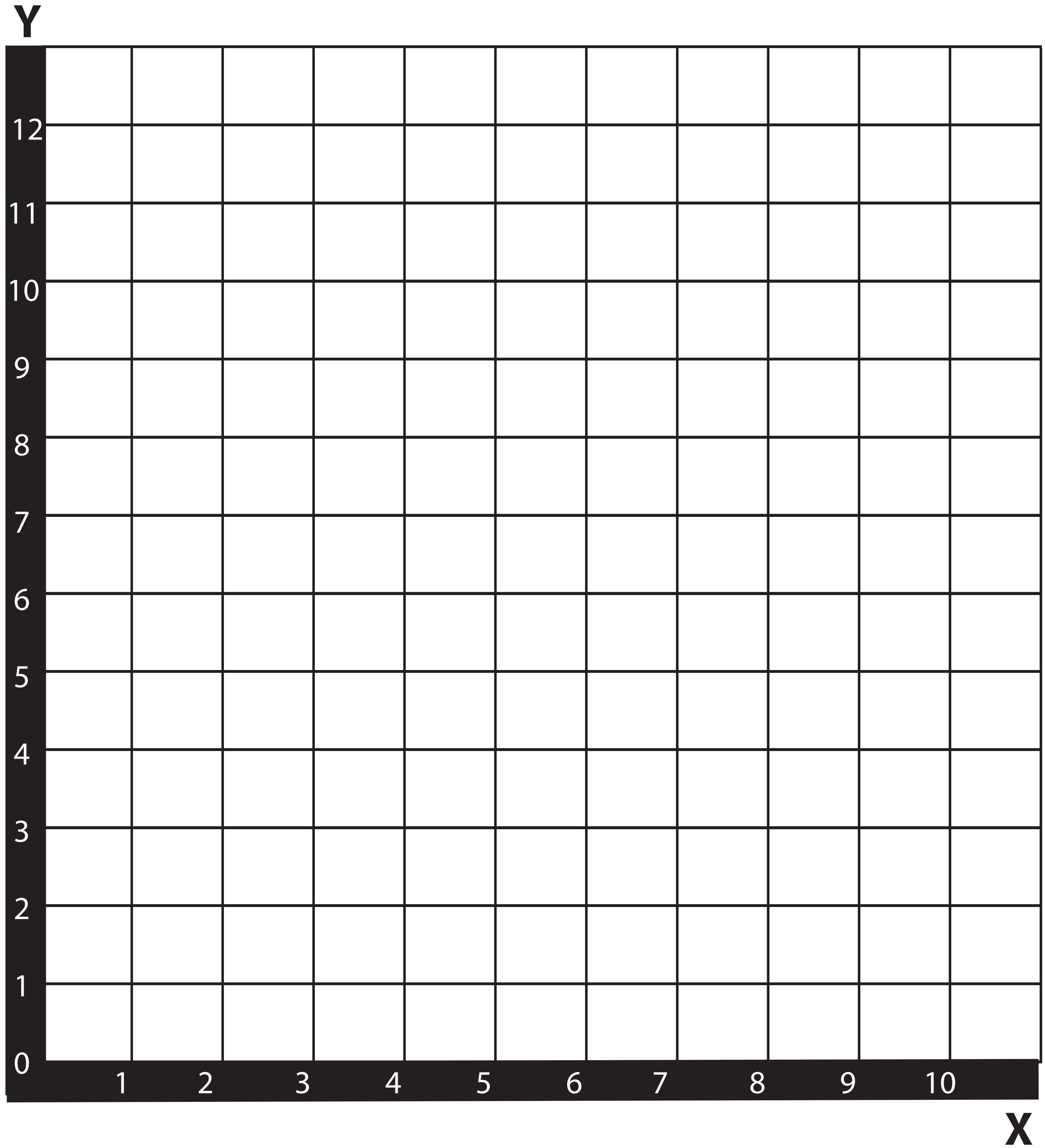


Over to X, up to Y, draw dot and connect.

(X,Y)	(X,Y)	(X,Y)	(X,Y)
(5,1)	(5,1)	(7,7)	(3,7)
(3,2)	(5,8)	(7,6)	(5,8)
(3,3)	(6,10)	(6,6)	(3,9)
(5,1)	(7,10)	(5,8)	(3,10)
(7,2)	(7,9)	(4,6)	(4,10)
(7,3)	(5,8)	(3,6)	(5,8)

X•Y

Connect the Dots



Over to X, up to Y, draw dot and connect.

(X,Y)	(X,Y)	(X,Y)
(5,7)	(5,1)	(5,7)
(4,8)	(8,4)	
(2,8)	(9,6)	
(1,6)	(8,8)	
(2,4)	(6,8)	

Connect-the-Dots: Pictures from Numbers.

Plotting points is simple. If you can count to 10 you can do it. You can use any graph paper to somplete this drawing or you can use a copy of the blank X•Y Connect-the-Dots worksheet found in this kit.

The two numbers in the parentheses determine the location of a point. The first number, the X-coordinate, is the distance of the point from the Y-axis. To find that position, count over on the X-axis from the origin that number of units to the right and stop.

The second number, the Y-coordinate, is the distance from the X-axis. Count up that number of units from the X-axis, keeping the same distance from the Y-axis. Make a dot at this point. You have just plotted a point or drawn a dot at the correct location.

Now plot the next point and connect it with a straight line to the last point. Proceed in this fashion with each pair of numbers, called coordinates, until the last pair. What do you see? Whatever it is you can thank your lucky numbers for it!

EASY

1. (2,2), (7,4),(7,7),(4,7),(2,2),(7,7),(7,4),(4,7)
2. (5,10),(1,5),(10,5),(5,10),(5,3),(1,3),(2,1),(9,1),(10,3),(5,3)

MEDIUM

1. (5,2),(4,5),(2,6),(4,7),(5,9),(6,7),(8,6),(6,5),(5,2)
2. (7,5),(4,5),(2,4),(5,4),(5,1),(5,4),(7,5),(7,2),(5,1),(2,1),(2,4)

HARD

1. (2,5),(1,6),(2,8),(3,9),(6,9),(7,8),(8,6),(9,8),(10,9),(9,5),(10,1),(9,2),(8, 4),(7,2),(6,1),(3,1),(2,2),(1,4),(2,5)
2. (1,3),(1,5),(2,6),(3,8),(4,8),(4,6),(2,6),(3,8),(4,9),(6,9),(6,6),(10,6), (10,3),(9,3),(9,2),(8,1),(7,1),(6,2),(6,3),(5,3),(5,2),(4,1),(3,1),(2,2), (2,3), (1,3)

X•Y

Multiplication Quiz

Mind	XY Chart	Calculator
3 X 5 =	8 X 7 =	4 X 9 =
6 X 7 =	6 X 9 =	2 X 7 =
9 X 2 =	3 X 2 =	6 X 7 =
6 X 6 =	7 X 5 =	9 X 3 =
3 X 7 =	8 X 6 =	6 X 8 =
4 X 3 =	7 X 9 =	2 X 4 =
9 X 5 =	5 X 5 =	7 X 2 =
3 X 8 =	6 X 2 =	5 X 4 =
3 X 6 =	9 X 8 =	2 X 8 =
7 X 2 =	6 X 4 =	7 X 7 =

X•Y

Multiplication Quiz

Mind	XY Chart	Calculator
3 X 5 =	8 X 7 =	4 X 9 =
6 X 7 =	6 X 9 =	2 X 7 =
9 X 2 =	3 X 2 =	6 X 7 =
6 X 6 =	7 X 5 =	9 X 3 =
3 X 7 =	8 X 6 =	6 X 8 =
4 X 3 =	7 X 9 =	2 X 4 =
9 X 5 =	5 X 5 =	7 X 2 =
3 X 8 =	6 X 2 =	5 X 4 =
3 X 6 =	9 X 8 =	2 X 8 =
7 X 2 =	6 X 4 =	7 X 7 =
8 X 6 =	7 X 7 =	4 X 4 =
6 X 8 =	6 X 5 =	7 X 5 =
9 X 3 =	3 X 7 =	7 X 8 =
6 X 2 =	7 X 4 =	9 X 3 =
3 X 9 =	8 X 3 =	6 X 1 =
4 X 6 =	6 X 8 =	9 X 4 =
9 X 5 =	5 X 4 =	7 X 3 =
7 X 8 =	6 X 7 =	9 X 7 =
3 X 6 =	8 X 4 =	2 X 8 =
9 X 2 =	6 X 6 =	8 X 7 =
Time :	Time :	Time ___________:___________

THE X•Y CHART ™ USER'S GUIDE

What is The X•Y Chart ™?

The X•Y Chart ™ is a colorful picture of mathematics.

It shows the 10 by 10 multiplication facts in their natural environment, a table of rows and columns cornered in the first quadrant of the rectangular coordinate system.

The large white X and Y letters, connected by the multiplication dot, tie the entire scene closely to algebra, more closely than the X and Y of the widened black X- and Y-axes.

The perfect squares are colored gray to highlight the symmetry of the 10 by 10 table and its visible commutative law of multiplication.

The bright primary colors of the quadrants signal a change of sign, a positive-negative switch.

The sections of the quadrants, which begin at the intersection of the X- and Y-axes, invite the observer to continue beyond the borders of the chart toward the limiting infinities, the outer bounds of the actual math quadrants.

The X•Y Chart ™ is designed to help teach multiplication in an effective visual manner.

Introduction

The X•Y Chart ™ is a colorful picture of mathematics. It unites the basic facts of multiplication with the visual system of higher math. The ability to visualize mathematics is fundamental to thinking mathematically. When math learners use the X•Y Chart ™ to learn multiplication and graphing, they will acquire a powerful image that provides them with the mental means to master math from kindergarten through college and beyond.

What follows is organized into three parts:
- reasons why you should show the large X•Y Chart ™ in the home and classroom,
- a series of activities to engage the math learner more closely with its contents, and
- a short mission statement.

Part I. Show the chart!

A. The chart creates a positive math environment

Display the X•Y Chart™. Its purpose is to communicate math to the observer, to foster a love for math, to prevent math "phobia". Math is not as difficult as some people seem to believe. The more the X•Y Chart™ is seen, the more its math image will become a part of the learner's mind and facilitate the understanding of important math concepts.

B. Highlight the importance of math

Hang the poster-size X•Y Chart™ in a prominent place in your home or classroom. Dedicating an area of valuable wall space to the chart demonstrates the importance of math. This also will help eliminate apathy towards math. Teachers in areas not usually associated with math (history, language arts, political science, etc.) can show that they too appreciate the central importance of mathematics by displaying the chart. Businesses interested in public education can do the same.

Part II. Discussion and Activities

Parents and teachers know that a good understanding of math can make a big difference in the intellectual development of our children. Success in formal education is greatly influenced by a student's attitude toward and ability in math. Here then are some activities that add to the natural visual learning of the X•Y Chart ™.

A. Numerals and Numbers

The symbol 6 is a numeral. The groups ****** and $$$$$$ and ♣♣♣♣♣♣ have 6 in common.

On the X•Y Chart™ products are represented by numerals as well as the number of squares within the rectangular area formed by the multiplication. Our visual grasp of magnitude or quantity forms our natural understanding of numbers. An understanding of the abstract representation of numbers by numerals is a higher level intellectual skill that we acquire when we first learn math.

The X•Y Chart™ presents multiplication in both forms, numbers and numerals.

Showing the math learner that the symbol 4 stands for the 4 squares in its rectangle will help the learner appreciate the difference between numerals and numbers.

B. Counting

Basic Counting

As soon as a child shows an interest in numbers, introduce the X•Y Chart™ and begin counting by 1. The simplest counting begins with 1 and continues by adding 1 to the previous number.

There are four lines within the X•Y Chart™ where counting by 1 makes an appearance. This is, after all, the most important sequence of numbers in mathematics:

1. the X-axis on the chart
2. the row of squares just above the X-axis
3. the Y-axis on the chart
4. the column of squares just to the right of the Y-axis

The simple act of counting from 1 to 10 and following along with the numbers in one of these four lines of the chart helps the child learn the numerals and see the ever-lengthening line representing or represented by the latest numeral. It also impresses in the mind the big picture of the chart in a way that passive observation wouldn't. This big picture will spark the child's natural interest in numbers.

Skip-Counting

After the child masters counting by 1, look at the numbers in any other row or column of the multiplication table. You will notice that each is a sequence of numbers that differ by the same number. When you counted by 1 you added 1 to each number. In the next row or column you begin with 2 and add 2 to each previous number. This pattern continues through 10.

Counting in this fashion, adding the same number to the last, is known as skip-counting and is preparation for multiplication**. Start with 2's – 2,4,6, 8, etc. Follow the row or column on the X•Y Chart™.** This is a little more challenging than counting by 1.

If you are in a class, have the students take turns saying the next number out loud. You can do the same at home. This makes counting more enjoyable and adds the spice of competition. Proceed by skip-counting by 5's, then by 10's, the easiest patterns. Then try it with 3,4,6,7,8, and 9.

When a child first starts skip-counting with the chart, the next numeral is the focus of attention, for that's the expected answer. It is important at this stage, however, to point out the increasing number of squares in the rectangle formed by the upper right corner where the numeral is placed and the lower left corner of the origin of the coordinate system. With each new number this rectangle grows by the same number of squares, the number of the skip-counting.

Repeated skip-counting lays a solid foundation for learning the multiplication facts. The numbers in the skip-counting are each products in the 10 x 10 table. This exercise also helps the child grasp the central mathematical concept that multiplication is the result of repeated addition of the same number. By using the X•Y Chart™ as a reference, its picture of mathematics settles ever more firmly into the mind of the math learner.

C. Multiplication as Repetitive Addition

The logical transition from skip-counting to multiplication is very easy, but often missed. If you were skip-counting by 5's, for example, you would begin by 5 for the first term, add 5 to reach 10 for the second term, add 5 to reach for 15 for the third term, etc. If you keep track of the number of the term you will see that this number "times" the number added equals the value for that term. Multiplication is this repeated addition. 5 + 5 + 5 = 15 is equivalent to 3 x 5 = 15.

For your math learner skip-counting is the more basic operation. **Practice skip-counting for all the numbers between 1 and 10 until it becomes second-nature, with no hesitations. Then introduce multiplication as repeated addition and show how skip-counting consists of these products.**

Multiplication and Memorization

Some modern math educators claim that memorization is not an effective way to learn math, that students are better off applying principles on a case-by-case basis. What are the practical consequences of this approach? At a recent event where children were taking multiplication tests for the pure joy of it, we noticed a child who did the following to calculate the product of 7 x 6:

7 x 6 = 6 + 6 + 6 + 6 + 6 + 6 + 6 = 12 + 12 + 12 + 6 = 24 + 18 = 42.

She wrote these numbers in columns but the above expression duplicates her method. This student clearly recognized that the multiplication sign meant repeated addition and applied this knowledge in a clever fashion to arrive at the answer. But did the student understand multiplication? In effect she did *not* multiply, she added.

Multiplication is an arithmetic operation that summarizes the more basic operation of repeated addition. To multiply the student must know the final sums of these repeated additions. <u>The applied arithmetic operation of multiplication is memorization!</u> Once you have mastered the 10 x 10 facts, or the single-digit facts, you can build on this knowledge to perform more complex multiplication such as 342 x 923. Problems like this require other rules for calculation, such as carrying digits and keeping position, but to get the process going the child has to memorize the single digit multiplication facts, not calculate them individually by adding.

Understanding Multiplication

As a way to emphasize the definition of multiplication as repeated addition, look at any product shown on the X•Y Chart™. Its numeral is located in the corner by the dot where the vertical line of the X value crosses the horizontal line of the Y value. These 2 lines form a rectangle with the X- and Y-axes. The origin of the coordinate system is the lower left corner of this rectangle, the dot where the numerical product appears is the upper right corner. Notice how the number of squares contained in the rectangle is represented by the product's numeral. (A quick look at the X•Y Chart™ might help you understand what you just read!)

Have the student count each square individually to prove that this is so or skip-count by either the length or the width of the rectangle until the product is reached. This growing rectangular area is a valuable picture for later math, especially in integral calculus where the area under a curve is important.

D. Connect the Dots

Do you remember drawing pictures by connecting numbered dots? This activity helps children learn to count. An even more valuable math experience can be gained if you first plot the points using the coordinates and then connect them.

Plotting points is simple. The child needs to be able to count by 1 and to know the meaning of "over and up." The accompanying X•Y Connect the Dot™ worksheets and the extra problem sets will produce simple figures. You can use any graph paper to complete this exercise or copy one of the X•Y Connect the Dot™ general work sheets.

Plotting the Dots

The two numbers contained in parentheses determine the location of a point. The first number, the X-coordinate, is the distance from the Y-axis. To find the dot's position, count over to the right and hold.

The second number, the Y-coordinate, is the distance from the X-axis. Count up that number from the X-axis, keeping the same distance from the Y-axis. You are now in the "over and up" position. Make a dot at this point where the two lines intersect.

Notice that the vertical line crosses the X-axis at the number of the first coordinate in the parentheses; the horizontal line crosses the Y-axis at the number of the second. Now plot the next point and connect the two dots with a straight line. Proceed in this fashion until you have plotted and connected all the dots and a picture emerges.

Welcome to the Visual World of Higher Math

Congratulations! You have successfully used the rectangular coordinate system invented by Rene Descartes to draw a picture. This great invention of the early 17th century brought together algebra and geometry for the first time. Now you see how these lifeless pairs of numbers transform into figures.

This graphing system happens to be the most important conceptual tool of higher math. Analytic geometry is built around it. Calculus uses it wherever it studies functions. Calculators employ it for graphing, computers and video games for animation and virtual reality. Often a source of math “phobia”, the coordinate system is actually so easy to use that children in kindergarten can master it in a matter of minutes. And they enjoy it.

As young children learn how to complete these connect-the-dot drawings, they master the basics of the rectangular coordinate system. This initial and positive exposure is critical to a child's math future. We have watched kids mesmerized by this activity for long periods, lost in deep concentration, experiencing the pleasure of serious math. After they have completed several of these connect-the-dot drawings, the X•Y Chart ™ will engage their attention even more since they now realize how the coordinates work. Its picture of mathematics will have taken root more firmly in their minds.

E. Multiplication of Coordinates

Now that your child has completed several connect-the-dots problems, it is time to learn multiplication using the **X•Y Chart ™.** Finding the product on the chart is no different than plotting a point. (We emphasize the "over and up" orientation since it ties nicely to the later math concept of a function working over a range of X values.)

Let's begin with this problem 3 x 5 = 15. The first number is the multiplier. It is the number in multiplication that represents the number of times the other number, or multiplicand, is added or how often its addition is repeated. If you multiply in order to calculate the total value of 3 nickels, then 3 is the multiplier and 5 the multiplicand.

Begin with the multiplier. Count "over" to 3 on the X-axis. From here count "up" to the second number's line, a horizontal line coming from the Y-axis at 5. You have arrived at the square containing the numeral 15, the product. This "over and up" method of multiplication for X times Y is the heart of the X•Y Chart ™.

You will see how the area of the rectangle is formed by the "over and up" movement and the answer is found at the final location. The product is not just this numeral, but also the area. Here we see clearly the symbolic, abstract answer, the numeral, as well as its concrete and visible representation as the squares of the rectangle's area. Descartes would appreciate this. His invention of the rectangular coordinate system united algebra and geometry and thus made it the tool of mathematics it has become.

Combining Connect-the-Dots and Multiplication with the X•Y Chart ™

An easy way to tie the connect-the-dots exercise with multiplication is to have the student multiply the coordinates of the dots just connected. If you make a transparency of the desktop size X•Y Chart the student can quickly check the results. This transparency will lie perfectly over the connect-the-dots sheets to show the products just below and to the left of the plotted dots.

F. Memorizing 100 Multiplication Facts -- Not Really.

The 10 x 10 multiplication problems and products do indeed make for 100 items to learn. However, take heart, thanks to some wonderful patterns you need to learn less than 25. Here's how:

1) Multiplication involves two numbers and an operation. It doesn't matter which number comes first. 3 x 5 has the same product as 5 x 3. This cuts the number of multiplication facts we need to learn almost in half, to 55. The perfect squares, the products of a number times itself, remain, since they show no difference, 5 x 5 is the same as 5 x 5. If you eliminated one you would be eliminating both. This is easier to see than to say. (This property of multiplication is known as the commutative law.)

2) Multiplication by 1 and by 10 is obvious. Subtract those products from the 55 and you now have only 36 facts to learn!

3) How about multiplying by 2, doubling the other number. With a little practice this is easy too. Subtract 8 more reducing the remaining facts to 28.

4) And the 5's. Counting by 5's has the advantage of our 5 fingers. This too can go. Subtract 5 more giving 23. **So much for learning 100!**

5) Multiplication by 9 has special qualities that make it easier to learn. First, notice that the sum of the two digits of each product adds to 9. Notice also that the first digit of each product is 1 less than the number multiplied by 9.

Finally, although this does not qualify as a mental math, you can use your hands to multiply by 9. Hold out all 10 fingers. Looking at your hands, count from the left the number of fingers as the number you wish to multiply by 9. Hide that finger. The number of fingers to the left of it form the ten's digit. The number of fingers to the right the units. 9 x 5, for example, leaves four fingers to the left of the left thumb and five fingers to the right. That makes 45, the product of 9 x 5.

6) The multiplication facts for the remaining numbers 3,4,6,7 and 8 will take a little more effort. (If you know any special tricks to learn these, please share them with us.)

G. Multiplication Quizzes: Mental, X•Y Chart ™, Calculator

Children love to race. The **X•Y Chart** ™ multiplication quizzes offer math learners a race with technology. The quizzes contained in this activity kit have three columns of multiplication problems taken from the 10 x 10 multiplication table that is the center of the **X•Y Chart** ™**.**

Time how long it takes to complete each column of problems. First complete the column labeled mental math using memory only. Record their completion time.

Next teach the system of finding the results using an **X•Y Chart** ™**.** Have either an **X•Y Chart** ™ desktop chart or postcard handy. With the "over and up" method complete each problem in the second column. Also record this time.

Finally, use a calculator to complete each problem. Writing the answers from memory is not permitted for this exercise. Mental math is cheating. Now record this time.

Very likely the fastest method will be mental, facts drawn effortlessly from memory. The **X•Y Chart** ™ method will finish second. The loser is the calculator. This surprises many students and teachers. Calculators aren't always the fastest way to calculate. Notice that the use of the **X•Y Chart** ™ is not only faster than the calculator, but it also reinforces the idea that multiplication is repeated addition and that the product is an expression of the rectangular area. The answer is served up as a numeral at the point of intersection and as an area within a rectangle. These represent two fundamental modes of mathematical expression.

III. Commitment to Communicate

Learning by Heart

Successful businesses learned long ago how to train customers without the customers knowing that they were being trained. Yet the customers learned very well. Children today have mastered the meanings of hundreds of commercial symbols before they begin to talk. No direct parent or teacher involvement is required to teach children the meaning of the Golden Arches, Coke™ or Beanie Babies™. Learning through advertising is largely a matter of repetitive visual stimulation.

This is possible because corporations spend billions of dollars on the presentation and the display of the learning materials. "Make it attractive and place it everywhere." These are the essence of effective advertising.

And yet, for a subject as valuable as mathematics, this "queen of the science", this language of God's creation, no comparable effort is made to instruct the masses. We spend more money and creative talent teaching the world to desire cars and french fries than we do teaching the world to understand math!

The X•Y Chart ™ Mission

The X•Y Chart™ will change this. It was developed in the belief that the primary methods of successful advertising, good presentation and constant repetition can be applied to the instruction of math.

In summary, it's all about a stubborn commitment to teach something that is inherently invaluable. This commitment has driven us to use the best means of communication available to teach math.

The X•Y Chart ™ Creator

His business card reads "SYSTEMS MATHEMATICS WRITING".

Professionally active in a field of applied mathematics, computer programming and systems analysis, X•Y Chart ™ creator Joe Speier finds much of his intellectual energy consumed by teaching high school algebra.

This father of 5 has tried to balance earning a living in the computer software field with designing educational posters and teaching math. The X•Y Chart™ is the result of a decade of wondering how to teach multiplication in an effective visual manner.

Joe is a native of Cincinnati, Ohio. He majored in German at Holy Cross College in Worcester, Massachusetts, taught a semester of grade school in Cincinnati, directed a youth program for four years in Berlin, Germany, taught high school for two years in Philadelphia and computer programming languages for three years at Miami University in Ohio.

Before turning his attention to helping students master basic math; Joe created the "How Are You Feeling Today?" chart, a visual tool to help people identify their feelings, illustrated by Pulitzer Prize winning editorial cartoonist Jim Borgman.